Annuity

Annuity : Annuity is a regular and equal payment which is made after a certain time span. Time span may be yearly , half yearly, quarterly, monthly etc.

Examples of Annuity :

→ Payment of housing loan, vehicle loan, life insurance premium etc.

Immediate Annuity

→ Immediate annuity or Annuity regular or ordinary annuity :- In this case payment/receipt takes place at the end of the period.

Important Formulas of Immediate Annuity:

(1) $A = \frac{c}{i}[(1 + i)^n - 1]$

(2) $P = \frac{c}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$

(3) $A = P\left(1 + \frac{r}{100}\right)^n$

$P\left(1 + \frac{r}{100}\right)^n$ = is called the factor of compounding.

(4) $P = \frac{A}{P(1 + \frac{r}{100})^n}$

$\frac{1}{P(1 + \frac{r}{100})^n}$ = is called the factor of discounting .

Annuity Due

→ **Annuity due or Annuity immediate:- In this case payment/receipt takes place at the beginning of the period.**

Important Formulas of Annuity Due:

(1) $A = \frac{c(1+i)}{i}[(1+i)^n - 1]$

(2) $P = \frac{c(1+i)}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right]$

A = Amount or Accumulated value or future value .

P = Present value or principal value .

C = Annuity .

i = $\frac{r}{100}$, r = rate of interest .

n = time (in years).

Question 1: Rs 400 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10^{th} payment? Given that $(1.005)^{10} = 1.0511$

Solution: Here c = Rs 400 (annuity or monthly invested value)
n = 10 (In the case of monthly compounding n is equal to number of months)
i= 6% per annum = 6/12%per month = 0.005 (because of monthly compounding r will be divide by 12) (i = $\frac{r}{100}$)
Future value of annuity after 10 month is given by

$$A(n,i) = c\left[\frac{(1+i)^n - 1}{i}\right]$$

$$A(10,0.005) = 400\left[\frac{(1+0.005)^{10} - 1}{0.005}\right]$$

$$= 400 \quad \left[\frac{1.0511 - 1}{0.005}\right]$$

$$= 400 \text{ x } 10.22$$

$$= \text{ Rs.4088}$$

Question 2 : To save for his son's college education. Mr. Avenendra decides to put Rs.12,000 aside at the end of every 6 months in a financial institution paying 8% interest compounded semiannually. If he begins this savings program when his son is 7 years old, find the amount of the investment by the time his son is 25 years old.(Given $(1.04)^{36} = 4.093$)

Solution: Here c = 12,000 , i = $\frac{4}{100}$ = 0.04 (r divide by 2 because of compounding semi-annually) , n = 18 x 2 = 36 (years multiplied by 2 because compounding half-yearly)

$\therefore$ Amount: $A = \frac{c}{i}[(1+i)^n - 1] = \frac{12,000}{0.04}[(1.04)^{36} - 1]$

$\therefore$ A= 3,00,000 [4.093 - 1] = 3,00,000 x (3.093) = Rs 9,27,900

Hence the amount of annuity is Rs 9,27,900

Question 3: Rs 1600 is invested at the end of each three months for two years in a financial institution paying interest 8% p.a. compounded quarterly. What is the accumulated value of this annuity ?

Solution: Here c = Rs 1600 (annuity or quarterly invested value)

n = 8 (In the case of quarterly compounding, years will be multiplied by 4)

i = $\frac{2}{100}$ = 0.02 (because of quarterly compounding r will be divide by 4) (i = $\frac{r}{100}$)

Future value of annuity after 10 month is given by

A(n,i) = c $\left[\frac{(1+i)^n - 1}{i}\right]$

A(8, 0.02) = 1600 $\left[\frac{(1+0.02)^8 - 1}{0.02}\right]$

= 1600 $\left[\frac{(1.02)^8 - 1}{0.02}\right]$ ⇒ 1600 $\left[\frac{1.171659381 - 1}{0.02}\right]$ ⇒ 1600 $\left[\frac{0.171659381}{0.02}\right]$

= 1600 x 8.5829690501

= Rs. 13,732.75

Question 4: A machine costs a company Rs. 30,000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of Rs 6,000 only. The price of the new model is estimated to be 20% higher than the price of the present one. find what amount should be set aside each every, out of the profits for the sinking fund, if it accumulates at 3.5% compounded annually[Given $(1.035)^{25} = 2.3632$]

Solution. Cost of the machine = 30,000 + $\frac{20}{100}$ x 30,000 = Rs (30,000 + 6,000) = Rs. 36,000

Scrap value of the old machine = Rs 6,000

Net cash required at the end of 25 years to purchase the new machine = 36,000 - 6,000 = Rs 30,000

Let c (annuity) be the amount which is set aside every years for the sinking fund

Here A = 30,000, i = $\frac{r}{100} = \frac{3.5}{100} = 0.035$, n = 25

$A = \frac{c}{i}[(1+i)^n - 1] \Rightarrow 30,000 = \frac{c}{0.035}[(1+0.035)^{25} - 1]$

$c = \frac{30,000 \times 0.035}{[(1+0.035)^{25} - 1]} = \frac{1050}{(2.3632 - 1)}$ = Rs. 770.2

Question 5: A machine costs a company Rs. 90,000 and its effective life is estimated to be 20 years. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of Rs 9,000 only. The price of the new model is estimated to be 20% higher than the price of the present one. find what amount should be set aside each every, out of the profits for the sinking fund, if it accumulates at 3.5% compounded annually[Given $(1.035)^{20} = 1.98978$]

Solution. Cost of the machine = 90,000 + $\frac{20}{100}$ x 90,000 = Rs (90,000 + 18,000) = Rs. 1,08,000

Scrap value of the old machine = Rs 9,000

Net cash required at the end of 20 years to purchase the new machine

= 1,08,000 -9,000 = 99,000

Let c (annuity) be the amount which is set aside every years for the sinking fund

Here A = 99,000, i = $\frac{r}{100} = \frac{3.5}{100} = 0.035$, n = 20

$$A = \frac{c}{i}[(1+i)^n - 1] \Rightarrow 99{,}000 = \frac{c}{0.035}[(1+0.035)^{20} - 1]$$

$$c = \frac{99{,}000 \times 0.035}{[(1+0.035)^{20} - 1]} = \frac{3465}{(1.98978886 - 1)} = \frac{3465}{(0.98978)} = 3500$$

Question 6: A refrigerator is purchased for Rs 100000 cash down and Rs 6000 per month for 3 years. Find the cash prices of the refrigerator if the payment includes interest at 15% compounded monthly. (Given $(1.0125)^{-36} = 0.6392$)

Solution. (as we know that payment is made monthly so it is the case of monthly compounding, and in the case of monthly compounding rate divide by 12 , years multiplied by 12)

Cash price of the refrigerator = amount of cash down payment + present value

Here Annuity (R) = Rs 6000 , Number of years (n) = 3 years = 3 x 12 = 36 ;

Rate of interest (i) = $\frac{15}{12 \times 100} = 0.0125$

Let P present value of the annuity . Then

$$P = c\left[\frac{1-(1+i)^{-n}}{i}\right] = \frac{6000}{0.0125}[1 - (1+0.0125)^{-36}]$$

$$= 480000[1 - (1.0125)^{-36}]$$

$$\Rightarrow P = 480000 \times [1 - 0.6392] = 480000 \times 0.3608 = \text{Rs } 1{,}73{,}184$$

Also amount of cash down payment = Rs 1,00,000

Cash price of the refrigerator = 1,73,184 + 1,00,000 = 2,73,184

Question 7: A T.V. is purchased for Rs 50,000 cash down and Rs 5,000 per month for 2 years. Find the cash prices of the refrigerator if the payment includes interest at 12% compounded monthly. (Given $(1.01)^{-24} = 0.7875661274$)

Solution. (as we know that payment is made monthly so it is the case of monthly compounding, and in the case of monthly compounding rate divide by 12 , years multiplied by 12)

Cash price of the T.V. = amount of cash down payment + present value

Here Annuity (R) = Rs 5,000 , Number of years (n) = 2 years = 2 x 12 = 24 ;

Rate of interest (i) = $\frac{12}{12 \, x \, 100}$ = 0.01

Let P present value of the annuity . Then

$P = c\left[\frac{1-(1+i)^{-n}}{i}\right] = \frac{5000}{0.01}[1 - (1 + 0.01)^{-24}] = 5{,}00{,}000[1 - (1.01)^{-24}]$

$\Rightarrow$ P = 5,00,000 x [1 - 0.7875661274] = 5,00,000 x 0.2124338726 = Rs 1,06,217

Also amount of cash down payment = Rs 50,000

Cash price of the refrigerator = 1,06,217 + 1,00,000 = 2,06,217