This theorem is very important for all competitive exams

Theorem: At the compound rate of interest a sum becomes n_1 times in x years (in time) and it becomes n_2 times in y years (in time), so in this case we apply the given relation:

$$(n_1)^{\frac{1}{x}} = (n_2)^{\frac{1}{y}}$$

Example 1: In compound interest a certain money becomes double in 4 years. At the same rate of interest in how many years it becomes 4times?

Solution : In this we apply the direct formula: $(n_1)^{\frac{1}{x}} = (n_2)^{\frac{1}{y}}$

$$\Rightarrow \quad (2)^{\frac{1}{4}} = (4)^{\frac{1}{y}}$$

$$\Rightarrow \quad (2)^{\frac{1}{4}} = (2)^{\frac{2}{y}}$$

Now equate the powers of both the sides as base is equal

$$\frac{1}{4} = \frac{2}{y} \Rightarrow y = 8$$

Example 2: In compound interest a certain money becomes double in 3 years. At the same rate of interest in how many years it becomes 4times?

Solution : In this we apply the direct formula: $(n_1)^{\frac{1}{x}} = (n_2)^{\frac{1}{y}}$

$$\Rightarrow \quad (2)^{\frac{1}{3}} = (4)^{\frac{1}{y}}$$

$$\Rightarrow \quad (2)^{\frac{1}{3}} = (2)^{\frac{2}{y}}$$

Now equate the powers of both the sides as base is equal

$$\frac{1}{3} = \frac{2}{y} \Rightarrow y = 6$$

Example 3:In compound interest a certain money becomes triple in 9 years.At the same rate of interest in how many years it becomes 9times?

Solution : In this we apply the direct formula: $(n_1)^{\frac{1}{x}} = (n_2)^{\frac{1}{y}}$

$$\Rightarrow \quad (3)^{\frac{1}{9}} = (9)^{\frac{1}{y}}$$

$$\Rightarrow \quad (3)^{\frac{1}{9}} = (3)^{\frac{2}{y}}$$

Now equate the powers of both the sides as base is equal

$\frac{1}{9} = \frac{2}{y} \Rightarrow y = 18$

Question 4: A sum of Rs 30,000 deposited at C.I becomes double after 4 years, In 20 years the sum will be _____ .

Solution: In this we apply the $(n_1)^{\frac{1}{x}} = (n_2)^{\frac{1}{y}}$

$$(2)^{\frac{1}{4}} = (x_2)^{\frac{1}{20}} \Rightarrow x_2 = (2)^{\frac{20}{4}} = 2^5$$

$x_2 = 32$ (At the same rate of interest the sum will becomes 32 times)

So total sum will be = 30,000 x 32 = 9,60,000

Question 5: A sum of Rs 18,000 deposited at C.I. becomes four times after 10 years, In 20 years the sum will be _____ .

Solution: In this we apply the $(n_1)^{\frac{1}{x}} = (n_2)^{\frac{1}{y}}$

$$(4)^{\frac{1}{10}} = (x_2)^{\frac{1}{20}} \Rightarrow x_2 = (4)^{\frac{20}{10}} = 4^2 = 16 \text{ times}$$

$x_2 = 16$ (At the same rate of interest the sum will becomes 16 times)

So total sum will be = 18,000 x 16 = 2,88,000

Question 6: A sum on C.I becomes three times in 7 years, in how many times at the same rate of interest the sum will become 27 times ?

Solution: $(3)^{\frac{1}{7}} = (27)^{\frac{1}{y}}$

$\Rightarrow (3)^{\frac{1}{7}} = (3)^{\frac{3}{y}} \Rightarrow$ y = 21 years

Question 7: A sum of Rs 5,000 deposited at C.I becomes double after 5 years, In 15 years the sum will be _____ .

Solution: In this we apply the $(n_1)^{\frac{1}{x}} = (n_2)^{\frac{1}{y}}$

$$(2)^{\frac{1}{5}} = (x_2)^{\frac{1}{15}} \Rightarrow x_2 = (2)^{\frac{15}{5}} = 2^3 = 8$$

$x_2 = 8$ (At the same rate of interest the sum will becomes 8 times)

So total sum will be = 5,000 x 8 = 40,000